

R^2 - corrections to chaotic inflation

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Abstract

Scalar density cosmological perturbations, spectral indices and reheating in a chaotic inflationary universe model, in which a higher derivative term is added, are investigated. This term is supposed to play an important role in the early evolution of the Universe, specifically at times closer to the Planck era.

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The most appealing cosmological scenario to date is the standard hot Big-Bang model. However, when this model is traced back to early times in the evolution of the Universe it presents some shortcomings (“puzzles”), such that the horizon, the flatness, the primordial density fluctuation problems, among others.

Inflation [1] has been proposed as a good approach for solving the cosmological “puzzles” mentioned above. The essential feature of any inflationary model proposed so far is the rapid but finite period of expansion that the Universe underwent at very early times in its evolution.

The present popularity of the inflationary scenario is entirely due to its ability to generate a spectrum of density perturbations which may lead to structure formation in the Universe. In this way, it has been mentioned that inflation is a good candidate for generating density fluctuations at scale significantly larger than the Hubble radius without breaking causality [2]. In essence the conclusion that all the observations of microwave background anisotropies performed so far [3] support inflation, rests on the consistency of the anisotropies with an almost Harrison-Zel’dovich power spectrum predicted by most of the inflationary universe models.

Among the different models we distinguish Linde’s chaotic inflationary scenario [4]. This model allows various initial distribution of the inflaton field, providing a wide class of theories where inflation occurs under quite natural initial conditions. In this respect, it is possible to consider a model in which it starts at time closer to the Planck time, $t_P \sim 10^{-43}$ sec, where, it is supposed that quantum corrections to the effective Lagrangian play an important role. It has been suggested that the effective gravitational Lagrangian closed to the Planck era should include higher derivative terms expressed for instance, as a function of the scalar curvature, R [5]. For simplicity, in this paper, we shall restrict to the case in which the effective action presents the R^2 -term only. Generalization to include other more complicated terms looks straightforward.

When the R^2 -term of the effective action is varied with respect to the metric tensor $g_{\mu\nu}$, it gives a term proportional to ${}^1H_{\mu\nu}$ in the field equations. This term coincides with that obtained as semiclassical limit of the interaction between a quantum matter field and gravity, via the expectation value of the corresponding energy-momentum tensor [6]. The

tensor ${}^1H_{\mu\nu}$ is given by

$${}^1H_{\mu\nu} = 2R_{;\mu;\nu} - 2g_{\mu\nu}R_{;\rho}{}^{;\rho} + 2RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^2.$$

Because higher order curvature terms are suppressed by the inverse Planck mass squared, their effects will be significant near the Planck era. In this respect, a higher derivative term like R^2 -term can be considered in addition to the Linde's chaotic inflationary universe. This kind of model has been treated in Refs.[7, 8]. In Ref.[7] the authors claim that the combined action of the R^2 term and the scalar field ϕ may lead to Double Inflation i.e., two consecutive inflationary stages separated by a power-law expansion. However the analysis of perturbations made in [7] is incomplete, because it was assumed that both contributions can be treated separately. These facts were elucidated in Ref.[8], where double inflation was found to occur under the following conditions: the mass of the scalar particle has to be small compared to the scalaron mass $m^2 \ll M^2$, and the vacuum polarization has to dominate initially. Otherwise the combined action of both effects leads to a single inflationary phase. They also found a break in the spectrum of perturbations (the so called Broken Scale Invariant-BSI) for double inflation.

The aim of the present paper is to study the effects that higher derivative term might have on inflation. Because (almost) all the literature on this topic works in the physical frame, leading to cumbersome expressions, we work here in the conformal frame where all the quantities can be easily derived. Here we study the single inflationary phase and left the double inflationary model for a future research [9].

Specifically, we find that inflation can be enlarged by the action of the R^2 term. Actually, we concentrate our analysis in the case of a single inflationary phase driven by both contributions. We find, in agreement with previous work [8], that this inflationary phase occurs for a special set of initial values of the fields, constrained by the parameters of the model. We also determine analytically the scalar density perturbations produced during inflation valid for a non-separable effective potential, as the one obtained in this model, which has not been discussed in the literature. By using the observational constraint for the amplitude of density perturbations we find that $m^2/M^2 \sim 10^{-1}$, showing consistency with a single inflationary phase. In Ref.[8] the authors use the ultrasynchronous gauge allowing them to treat the problem in the physical frame, where it is found that longitudinal potentials are proportional to the ultrasynchronous potential. Here we will consider longitudinal gauge

quantities, which are known to be gauge invariants [10]. Moreover, we obtain explicit analytical expressions for the spectral indices when the universe leaves the horizon, showing small deviations from the Harrison-Zel'dovich spectrum, that can be observationally interesting. We also perform in this model the analysis of reheating, based on the modern theory of preheating.

We consider the effective action given by

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{R^2}{6M^2} \right) + \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) \right] \quad (1)$$

where R , as above, represents the scalar curvature, ϕ is the “inflaton” scalar field, $V(\phi)$ is the inflaton potential which drives inflation, and M is an arbitrary parameter with dimension of Planck mass, $m_P \sim 10^{19} \text{ GeV}$ and $\kappa^2 = 8\pi/m_P^2$.

In order to calculate the scalar density perturbation we take the following conformal transformation [12]

$$\bar{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu}, \quad (2)$$

with

$$\Omega^2(x) = \left(1 + \frac{R}{3M^2} \right), \quad (3)$$

where $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ represent the original and the transformed metric, respectively. Introducing a new field ψ defined by

$$\psi = \frac{1}{\kappa\beta} \ln \left(1 + \frac{R}{3M^2} \right), \quad (4)$$

where $\beta \equiv \sqrt{2/3}$, and substituting eqs. (2-4) in the action (1), it becomes a Hilbert-Einstein-type action given by

$$\bar{S} = \int d^4x \sqrt{-\bar{g}} \left\{ \frac{\bar{R}}{2\kappa^2} + \frac{1}{2}(\partial\psi)^2 + \frac{1}{2}e^{-\beta\kappa\psi}(\partial\phi)^2 + \right. \\ \left. -\tilde{V}(\phi, \psi) \right\}, \quad (5)$$

where the potential $\tilde{V}(\phi, \psi)$ takes the form

$$\tilde{V}(\phi, \psi) = \frac{3M^2}{4\kappa^2} (1 - e^{-\beta\kappa\psi})^2 + e^{-2\beta\kappa\psi} V(\phi). \quad (6)$$

Notice that, if we disregard the inflaton field ϕ , then inflation is described entirely by gravity, since the model becomes supported by the R^2 -theory [13].

Let us introduce into the action (5) the conformal Friedmann-Robertson-Walker (FRW) metric

$$d\bar{s}^2 = d\bar{t}^2 - \bar{a}(\bar{t})^2 dx_i dx^i, \quad (7)$$

where $dx_i dx^i$ and $\bar{a}(\bar{t})$ represent the flat three-surface and the conformal scale factor, respectively. Considering also that the corresponding scalar fields ϕ and ψ are homogeneous, i.e., $\phi = \phi(\bar{t})$ and $\psi(\bar{t})$, then, after variations of this action respect to the different field variables, the following Equations of motion are found

$$3 \bar{H}^2 = \kappa^2 \left[\frac{1}{2} \dot{\psi}^2 + \frac{1}{2} e^{-\beta \kappa \psi} \dot{\phi}^2 + \tilde{V}(\psi, \phi) \right],$$

$$\ddot{\psi} + 3 \bar{H} \dot{\psi} = -\frac{\beta \kappa}{2} e^{-\beta \kappa \psi} \dot{\phi}^2 - \frac{\partial \tilde{V}(\psi, \phi)}{\partial \psi}, \quad (8)$$

and

$$\ddot{\phi} + 3 \bar{H} \dot{\phi} = \beta \kappa \dot{\psi} \dot{\phi} - e^{\beta \kappa \psi} \frac{\partial \tilde{V}(\psi, \phi)}{\partial \phi},$$

where the dots denote differentiation respect to the time \bar{t} and $\bar{H} = \dot{\bar{a}}/\bar{a}$ defines the Hubble parameter in the conformal frame.

In the slow-roll over approximation the field eqs. (8) become,

$$3 \bar{H}^2 = \kappa^2 \tilde{V}(\psi, \phi),$$

$$3 \bar{H} \dot{\psi} = -\frac{\partial \tilde{V}(\psi, \phi)}{\partial \psi}, \quad (9)$$

and

$$3 \bar{H} \dot{\phi} = -e^{\beta \kappa \psi} \frac{\partial \tilde{V}(\psi, \phi)}{\partial \phi}.$$

This set of eqs. is valid if the following conditions are met

$$\max\{\dot{\psi}^2, e^{-\beta \kappa \psi} \dot{\phi}^2\} \ll \tilde{V}(\psi, \phi),$$

$$e^{\beta \kappa \psi} (\tilde{V}(\psi, \phi)_{,\phi})^2 \ll \tilde{V}(\psi, \phi) |\tilde{V}(\psi, \phi)_{,\psi}|,$$

and

$$e^{-\beta \kappa \psi} \tilde{V}(\psi, \phi)_{,\phi\phi} \ll |\tilde{V}(\psi, \phi)_{,\psi}|,$$

where $\tilde{V}(\psi, \phi)_{,\phi}$ and $\tilde{V}(\psi, \phi)_{,\psi}$ represent the derivatives of the potential $\tilde{V}(\psi, \phi)$ respect to the fields ϕ and ψ , respectively. A solution to the equation of motion (9) for the chaotic inflationary potential, $V(\phi) = \frac{1}{2} m^2 \phi^2$ [4] and $M^2 \neq 2m^2$ is given by

$$e^{\beta \kappa \psi} = \frac{\beta^2 \kappa^2 m^2}{(M^2 - 2m^2)} \phi^2 + 1 \approx \frac{\beta^2 \kappa^2 m^2}{(M^2 - 2m^2)} \phi^2, \quad (10)$$

where, we have taken the integration constant $C = 0 \implies e^{\beta\kappa\psi_o} - 1 = \frac{\beta^2\kappa^2 m^2}{(M^2 - 2m^2)} \phi_o^2$. The subscript 0 denotes the values of each field at the beginning of inflation. The approximation involve in (10) is used for simplicity. In this case, we find

$$\phi^2 = \phi_o^2 - \frac{(M^2 - 2m^2)}{\bar{H}_o \kappa^2} \bar{t} \quad (11)$$

and

$$\psi = \frac{1}{\beta\kappa} \ln \left[e^{\beta\kappa\psi_o} - \frac{2m^2}{3\bar{H}_o} \bar{t} \right], \quad (12)$$

where

$$\bar{H}_o^2 = \frac{M^2}{4} (1 - e^{-\beta\kappa\psi_o})^2 + \frac{m^2 \kappa^2 \phi_o^2}{6} e^{-2\beta\kappa\psi_o} = \frac{\kappa^2 \tilde{V}_o}{3}.$$

Since we are mainly interested in the existence of inflationary stage, we should take the case in which $M > \sqrt{2}m$. Here, the shape of the potential exhibits an appropriated form for inflation to occur. This is not the case if $M \leq \sqrt{2}m$, since the potential presents an unacceptable peak for certain values of the scalar fields, which would force us to restrict strongly the range of these fields for inflationary models to occur.

The period of the superluminal expansion can be determined by the value of the inflaton field ϕ where the slow-rollover approximation breaks down. This happens when $|\ddot{\phi}| \simeq |3\bar{H}\dot{\phi}|$. This condition gives

$$\phi^2(\bar{t}_e) = \phi_e^2 \simeq \frac{(M^2 - 2m^2)}{6\bar{H}_o^2 \kappa^2}, \quad (13)$$

and the inflationary period results in

$$\bar{t}_e \simeq \frac{\bar{H}_o \kappa^2 \phi_o^2}{(M^2 - 2m^2)} - \frac{1}{6\bar{H}_o}, \quad (14)$$

where the subscript e denotes the values at the end of inflation.

The requirement for solving some of the puzzles present in standard Big-Bang cosmology is

$$\frac{a(t_e)}{a(0)} = \frac{\bar{a}(\bar{t}_e)}{\bar{a}(0)} e^{\beta\kappa/2(\psi_o - \psi(\bar{t}_e))} \gtrsim e^{65}. \quad (15)$$

This results in a condition for the initial value of the inflaton field given by

$$9\beta^2 \bar{H}_o^2 \phi_o^2 \kappa^2 \gtrsim 8^2 [M^2 - 2m^2] \quad (16)$$

or

$$\kappa^2 \phi_o^2 \gtrsim \frac{[M^2 - 2m^2]}{M^2} \left[46 - \frac{M^2}{\beta^2 m^2} \right], \quad (17)$$

which depends on the parameters M and m , as we could expect for two-field models with a no separable potential.

In order to calculate the scalar density fluctuations, we follow a similar approach to that described in ref. [14]. We take the perturbed metric in the longitudinal gauge, which is known to be manifestly gauge-invariant [10]. Therefore, in comoving coordinates we write

$$d\bar{s}^2 = (1 + 2\bar{\Phi}) d\bar{t}^2 - \bar{a}^2(1 - 2\bar{\Psi}) \delta_{ij} d\bar{x}^i d\bar{x}^j, \quad (18)$$

where the scalar metric perturbation fields are functions of the space and time coordinates, i.e., $\bar{\Phi} = \bar{\Phi}(\vec{x}, \bar{t})$ and $\bar{\Psi} = \bar{\Psi}(\vec{x}, \bar{t})$.

In order to describe a linear theory for the cosmological perturbations, we introduce fluctuations into the scalar fields ϕ and ψ , so that we write $\phi(\vec{x}, \bar{t}) = \phi(\bar{t}) + \delta\phi(\vec{x}, \bar{t})$ and $\psi(\vec{x}, \bar{t}) = \psi(\bar{t}) + \delta\psi(\vec{x}, \bar{t})$, where the background fields, $\phi(\bar{t})$ and $\psi(\bar{t})$, are solutions to the homogeneous Einstein equations, and the deltas are small perturbations that represent small fluctuations of the corresponding scalar fields.

Since the spatial part of the resultant energy-momentum tensor becomes diagonal, then it found that $\bar{\Phi} = \bar{\Psi}$. With this result, we can write the perturbed Einstein field equations, in which we can take spatial Fourier transform for the different variables. It is found that each mode satisfies the corresponding linearized perturbed field equation, given by

$$\dot{\bar{\Phi}} + \bar{H}\bar{\Phi} = \frac{\kappa^2}{2} \left[e^{-\beta\kappa\psi} \dot{\phi} \delta\phi + \dot{\psi} \delta\psi \right], \quad (19)$$

$$\begin{aligned} \delta\ddot{\phi} + (3\bar{H} - \beta\kappa\dot{\psi})\delta\dot{\phi} + \left(\frac{k^2}{\bar{a}^2} + e^{\beta\kappa\psi} \tilde{V}_{,\phi\phi}\right)\delta\phi \\ - \beta\kappa\dot{\phi}\delta\dot{\psi} + (e^{\beta\kappa\psi} \tilde{V}_{,\phi})_{,\psi}\delta\psi = \\ 2(\ddot{\phi} + 3\bar{H}\dot{\phi})\bar{\Phi} + 4\dot{\phi}\dot{\bar{\Phi}} - 2\beta\kappa\dot{\phi}\dot{\psi}\bar{\Phi}, \end{aligned} \quad (20)$$

and

$$\begin{aligned} \delta\ddot{\psi} + 3\bar{H}\delta\dot{\psi} + \left(\frac{k^2}{\bar{a}^2} - \frac{\beta^2\kappa^2}{2} e^{-\beta\kappa\psi} \dot{\phi}^2 + \tilde{V}_{,\psi\psi}\right)\delta\psi \\ + \beta\kappa e^{-\beta\kappa\psi} \dot{\phi} \delta\dot{\phi} + \tilde{V}_{,\psi\phi} \delta\phi \\ = 2(\ddot{\psi} + 3\bar{H}\dot{\psi})\bar{\Phi} + 4\dot{\psi}\dot{\bar{\Phi}} + \beta\kappa e^{-\beta\kappa\psi} \dot{\phi}^2 \bar{\Phi}. \end{aligned} \quad (21)$$

If in this set of equations we neglect terms proportional to $\bar{\Phi}$ and two time derivatives, and we consider non-decreasing adiabatic and isocurvature modes on large scale, i.e. $k \ll \bar{a}\bar{H}$, the following equations are obtained [15]

$$\bar{\Phi} = \frac{\kappa^2}{2\bar{H}} \left[e^{-\beta\kappa\psi} \dot{\phi} \delta\phi + \dot{\psi} \delta\psi \right], \quad (22)$$

$$3\bar{H}\delta\dot{\phi} + (e^{\beta\kappa\psi}\tilde{V}_{,\phi})_{,\psi}\delta\psi + e^{\beta\kappa\psi}\tilde{V}_{,\phi\phi}\delta\phi + 2e^{\beta\kappa\psi}\tilde{V}_{,\phi}\bar{\Phi} = 0, \quad (23)$$

and

$$3\bar{H}\delta\dot{\psi} + \tilde{V}_{,\psi\psi}\delta\psi + \tilde{V}_{,\psi\phi}\delta\phi + 2V_{,\psi}\bar{\Phi}, \quad (24)$$

where \tilde{V} is the potential specified by eq. (6).

For $V(\phi) = m^2\phi^2/2$ we can solve the above equations, following a similar approach to ref. [15]. It is found that

$$\frac{\delta\phi}{\dot{\phi}} = \frac{1}{\bar{H}} [C_1 + C_2 - C_2 f_1(\psi, \phi)], \quad (25)$$

$$\frac{\delta\psi}{\dot{\psi}} = \frac{1}{\bar{H}} [C_1 + C_2 f_2(\psi, \phi)] \quad (26)$$

and

$$\bar{\Phi} = -C_1 \frac{\dot{H}}{\bar{H}^2} + \frac{C_2}{\kappa^2} \left(f_2 \frac{\tilde{V}_{,\psi}^2}{V^2} + \frac{\tilde{V}_{,\phi}^2}{\tilde{V}^2} e^{\beta\kappa\psi} (1 - f_1) \right), \quad (27)$$

where

$$f_1 = f_1(\psi, \phi) = \alpha_1(\phi_e, \psi_e) + e^{-\beta\kappa\psi} \\ \times \left[C_{e2} \left(\frac{3M^2}{4\kappa^2} (2 - e^{-\beta\kappa\psi}) - \frac{m^2\phi^2}{2} e^{-\beta\kappa\psi} \right) - 2C_{e1} \right],$$

and

$$f_2 = f_2(\psi, \phi) = \alpha_2(\phi_e, \psi_e) + \left[\frac{3M^2}{(3M^2(e^{-\beta\kappa\psi} - 1) + 2\kappa^2 m^2 \phi^2 e^{-\beta\kappa\psi})} \right] \\ \times \left[C_{e2} \left(\frac{m^2}{2} \phi^2 e^{-\beta\kappa\psi} - \tilde{V} \right) \ln \left(\frac{3M^2(e^{-\beta\kappa\psi} - 1)}{4\kappa^2} + \frac{m^2\phi^2 e^{-\beta\kappa\psi}}{2} \right) + C_{e1}(e^{-\beta\kappa\psi} - 1) \right]$$

where α_1 and α_2 are arbitrary constants and C_{e1} and C_{e2} are given by $C_{e1} = \frac{e^{-\beta\kappa\psi_e}}{\tilde{V}_e} \left[\frac{3M^2}{4\kappa^2} (1 - e^{\beta\kappa\psi_e}) + \frac{m^2\phi_e^2}{2} \right]$ and $C_{e2} = \frac{1}{\tilde{V}_e}$, respectively.

In the expressions (25-27), terms in proportion to C_1 and C_3 represent adiabatic and isocurvature modes, respectively.

From eqs. (25) and (26) the curvature perturbation due to primordially adiabatic fluctuation at the end of inflation, when the scale crosses the horizon is given by

$$\frac{\delta\bar{\rho}}{\bar{\rho}} \approx \bar{H}_h \left[\frac{\delta\psi}{\dot{\psi}} [1 - f(\psi, \phi)] + \frac{\delta\phi}{\dot{\phi}} f(\psi, \phi) \right]_h, \quad (28)$$

where

$$f(\psi, \phi) = \frac{f_2}{[f_1 + f_2 - 1]}.$$

In the limit $M^{-1} \rightarrow 0 \Rightarrow \psi = 0$, which gives $f = Cte.$ and thus we obtain the standard result for the density perturbation $\delta\bar{\rho}/\bar{\rho} \propto \bar{H}^2/\dot{\phi}$. The magnitude of the density perturbations take the form

$$\begin{aligned} \frac{\delta\bar{\rho}}{\bar{\rho}} &\approx \frac{\beta\kappa^3\bar{H}_o^3\phi_h^2}{[M^2 - 2m^2]} \\ &\times \left[1 + f_{\phi_h} \left(\frac{2m}{\sqrt{(M^2 - 2m^2)}} - 1 \right) \right], \end{aligned} \quad (29)$$

where ϕ_h represent the field scalar ϕ at which the universe crosses the Hubble radius and f_{ϕ_h} is the function f evaluate at ϕ_h . Since $\bar{a}(\bar{t}_h)/\bar{a}(\bar{t}_e) = k\bar{H}_o^{-1}$ we find

$$\phi_h^2 = \frac{(M^2 - 2m^2)}{\bar{H}_o^2\kappa^2} \left[\ln(\bar{H}_ok^{-1}) + \frac{1}{6} \right], \quad (30)$$

and in view that the log term is of the order of $\mathcal{O}(10^2)$ and $\delta\bar{\rho}/\bar{\rho} \sim 10^{-5}$, we find, from eqs. (29) and (30), the following order for the parameters of our model $m^2/M^2 \sim 10^{-1}$. For this order the initial value of ϕ becomes restricted from below $\kappa\phi_o \gtrsim 5$.

Another interesting quantity to investigate is the spectral index n_S of the power spectrum of primordial fluctuations, which becomes specified by $P(k) \propto k^{n_S}$. It is well known that most of the inflationary scenarios are compatible with an almost Harrison-Zel'dovich spectrum, i.e., $n_S \approx 1$, and thus, it provides detailed information about the early time in the evolution of the universe. This prediction of the inflationary universe models is sustained by the COBE data [3], which are statistically compatible with the $n_S \approx 1$ result.

In our model, we could obtain the spectral index by using the following standard expression

$$\bar{n}_S - 1 = \frac{d \ln \left| \frac{\delta\bar{\rho}}{\bar{\rho}} \right|^2}{d \ln k}, \quad (31)$$

where, together with eqs. (29) and (30), it is obtained,

$$\bar{n}_S - 1 = \frac{-2}{[\ln(\bar{H}_ok^{-1}) + \frac{1}{6}]} + f'_k \left[\frac{(f_k - 1) + \frac{m^2}{(M^2 - 2m^2)} f_k}{(1 - f_k)^2 + \frac{m^2}{(M^2 - 2m^2)} f_k^2} \right]$$

where f_k represents the function f expressed in terms of k and $f'_k = df_k/d\ln k$. The right-hand-side of the latter equation represents the deviation from the $\bar{n}_S = 1$ Harrison–Zel’dovich spectrum. For instant, if we use the constraint obtained above, i.e., $m^2/M^2 \sim 10^{-1}$, we obtain that $\bar{n}_S \approx 0.98$, which differs from the Harrison–Zel’dovich (HZ) value approximately in a 2%. We should note that this deviation is inside of the observational range, since it has been reported that the scalar perturbations do not differ from scale invariant by a large amount, since $|n_S - 1| \sim \mathcal{O}(0.1)$, i.e., inside of the 10% [20].

In addition to the scalar curvature perturbations there are tensor or gravitational wave perturbations. These perturbations can also be generated from quantum fluctuations during inflation. Since we have chosen to work in the Einstein conformal frame we can use the standard results for the evolution of tensor perturbations. The spectrum of the gravitational wave P_G is known to be given by $P_G = 8\kappa^2(\bar{H}/2\pi)^2$, and where the spectral index is defined by $\bar{n}_G = d\ln P_G/d\ln k$ [10, 21]. Thus, we find in our case

$$\bar{n}_G = \left[\frac{3M^4}{4\kappa^4 \bar{H}_o^2 \beta} \ln(\bar{H}_o k^{-1}) - 1 \right]^{-1} \left[\ln(\bar{H}_o k^{-1}) + \frac{1}{6} \right]^{-1}$$

where $\beta = 9M^2(M^2 - 2m^2)/(8\kappa^4 m^2)$. We could give an order of magnitude of this parameter if we use the value obtained for the quotient m/M determined above. Our result gives $\bar{n}_G \approx -0.22$. Notice that this value differs in about a 20% of that value predicted by the HZ spectrum which gives $\bar{n}_G \sim 0$. Thus, our model presents a significant deviation from the HZ result. Only astronomical observations will tell us which spectrum will be the right one.

Note that every expression was determined in the Einstein conformal frame. In order to see what happens with the same expressions but in the physical frame, we have to use the original conformal transformation eqs. (2) and (3). The quantities in the Einstein frame are related to the physical frame by means of a conformal transformation given by eq.(2). In both frame, the gauge-invariant perturbations are related by [22]

$$\Phi = \bar{\Phi} - \frac{\delta\Omega}{\Omega}, \quad (32)$$

where from eqs. (3) and (4) it is found that $\delta\Omega = \beta \Omega \delta\psi/2$, with $\delta\psi \sim \bar{H}$.

In the physical frame, the gravitational potential becomes

$$\Phi = C_1 \left(1 - \left[H + 2\frac{\Omega_{,t}}{\Omega} \right] \frac{1}{a\Omega^2} \int^t a\Omega^2 dt' \right). \quad (33)$$

Comparing both expressions, it is observed that they are very different, but, during inflation, where the Hubble parameter remains practically constant, we expect that the quadratic

scalar curvature term in the effective Lagrangian varies slowly, so that the Ω conformal factor could be considered constant during this period. Therefore, the adiabatic fluctuations become described by the same expression in both frames. The same argument can be drawn for the spectral indices \bar{n}_S and \bar{n}_G , since it depends directly of the expression for $\delta\bar{\rho}/\bar{\rho}$ and \bar{H} respectively.

Every inflationary model needs a mechanism to reheat the universe, i.e., a physical mechanism capable to patch the inflationary era with the radiation dominated phase (needed for example in nucleosynthesis). The current understanding of this process comprises three phases: preheating (the resonant amplification phase), the perturbative decay and thermalization [11]. Reheating in the R^2 model has been considered by Suen and Anderson [23] by solving the semiclassical backreaction equations. Recently Tsujikawa *et al.* [24] have studied the preheating phase with nonminimally coupled scalar fields. However, as far as we know the study of preheating in the model that we are considering here, has not been discussed in the literature.

In order to study the preheating phase we consider the inflaton field coupled with another scalar field, say χ through the interaction term

$$-\frac{1}{2}g^2\phi^2\chi^2, \quad (34)$$

where g is the coupling constant. During the coherent oscillations of the inflaton field around the minimum at the origin, which is precisely the period where reheating occurs, the scalaron field moves towards the minimum, but in a time scale shorter than that of the inflaton ϕ . In essence we can consider it nearly constant during preheating; which is the first part of reheating. Under these assumptions the inflaton field equation reads

$$\ddot{\phi} + 3\bar{H}\dot{\phi} = -e^{\beta\kappa\psi}(m^2 + g^2\langle\chi^2\rangle)\phi, \quad (35)$$

where we have replaced χ^2 by the quantum expectation value, $\langle\chi^2\rangle$, doing manifest the mean field approximation involved. The precise definition of this object will be given in short. Furthermore, the equation of the χ field is expressed by

$$\ddot{\chi} + 3\bar{H}\dot{\chi} - \nabla^2\chi = -e^{\beta\kappa\psi}(g^2\phi^2)\chi. \quad (36)$$

As we can notice here, the preheating phase proceeds very similar to the standard way, because the only difference is the exponential damping term in the coupling. By considering

the field χ as a quantum operator we can expand it as

$$\chi = \int \frac{d^3k}{(2\pi)^{3/2}} \left[a_k \chi_k(\bar{t}) e^{-i\mathbf{k}\mathbf{x}} + a_k^\dagger \chi_k^*(\bar{t}) e^{i\mathbf{k}\mathbf{x}} \right], \quad (37)$$

where the a 's are annihilation and creation operators, and the mode functions $\chi_k(t)$ satisfy the equation

$$\ddot{\chi}_k + 3\bar{H}\dot{\chi}_k + \left[\left(\frac{k}{\bar{a}} \right)^2 + e^{\beta\kappa\psi} g^2 \phi^2 \right] \chi_k = 0. \quad (38)$$

The expectation value of χ^2 is expressed in terms of the mode functions as

$$\langle \chi^2 \rangle = \frac{1}{2\pi^2} \int dk k^2 |\chi_k^2|. \quad (39)$$

Eq. (38) can be rewritten as a harmonic oscillator with a time dependent frequency

$$\ddot{X}_k + \Omega_k^2(\bar{t}) X_k = 0, \quad (40)$$

where we have defined the variable $X_k = \bar{a}(\bar{t})^{2/3} \chi_k$ and the frequency is given by

$$\Omega_k^2 = \left(\frac{k}{\bar{a}} \right)^2 + e^{\beta\kappa\psi} g^2 \phi^2 - \frac{3}{4} \bar{H}^2 - \frac{3}{2} \frac{\ddot{\bar{a}}}{\bar{a}}. \quad (41)$$

Usually the last two terms in (41) are of the same order and also very small compare with the effective mass m_χ , so we can neglect these terms as a first approximation. In this case the frequency depends on the scale factor $\bar{a}(\bar{t})$, the scalar field $\phi(\bar{t})$ and $\psi(\bar{t})$.

During the oscillations of the inflaton field ϕ , the scalaron ψ first rolls towards the minimum and after that start to oscillate around it, right after a few oscillations of ϕ . Preheating occurs here before ψ start to oscillate. If we define $\bar{\psi}_0$ as the value at this point, an approximated solution of (35) is

$$\phi(\bar{t}) \simeq \Phi(\bar{t}) \sin(\omega \bar{t}), \quad (42)$$

where $\omega = e^{\beta\kappa\bar{\psi}_0} m$ and the amplitude $\Phi(\bar{t})$ changes slowly with \bar{t} . Inserting this solution in eq. (41) we find a Mathieu type equation. It controls both the narrow and broad resonance particle production. We have performed numerical studies of that equation finding no surprises, just the evident effect of damping terms in (35) and (40). After ψ start to oscillate (for $\psi < \bar{\psi}_0$), the system becomes completely dominated by ψ , implying another period of reheating, this time without any resonance amplification, in which the ψ field decays. A detailed research of preheating in these models, considering both inflationary realizations, is under study [9].

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